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AUTHOR	Weathersby, George B.; Trueheart, William E.
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ABSTRACT

Many colleges and universities have claimed some degree of financial distress, have sought increased federal, state, and philanthropic assistance, and have agonized over predicted declines in youth demand for higher education. In response to these concerns, numerous cost studies have been conducted but little is known of the relative efficiency of resource use in higher education. This paper estimates empirically the resource use behavior of relatively efficient, historically black four-year colleges and shows that for the variables considered, there are significant economies achievable by adopting the resource use patterns of relatively efficient institutions. Focus is on the theory of production function analysis, previous production function studies in higher education, and its use in black colleges. A five-page bibliography is included. (Author/LBH)

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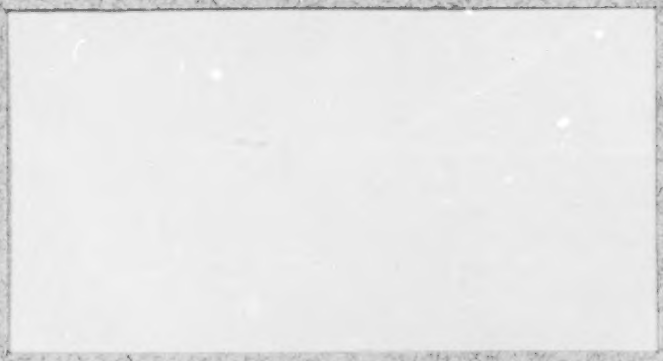

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The Development of Colleges and Universities

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PRODUCTION FUNCTION ANALYSIS IN HIGHER EDUCATION:

GENERAL METHODOLOGY AND APPLICATIONS TO FOUR YEAR BLACK COLLEGES

William E. Trueheart
George B. Weathersby

Appendix G

The Development of Institutions of Higher Education;
Theory and Assessment of Impact of Four Possible Areas
of Federal Intervention

ABSTRACT

Many colleges and universities have claimed some degree of financial distress, have sought increased federal, state, and philanthropic assistance, and have agonized over predicted declines in youth demand for higher education. In response to these concerns numerous cost studies have been conducted, but little is known of the relative efficiency of resource use in higher education. In this paper, we estimate empirically the resource use behavior of relatively efficient historically black four year colleges and show that for the variables considered, there are significant economies achievable by adopting the resource use patterns of relatively efficient institutions.

TABLE OF CONTENTS

	Page
List of Illustrations	ii
List of Tables	iii
I. INTRODUCTION	1
II. THEORY OF PRODUCTION FUNCTION ANALYSIS	3
III. PREVIOUS PRODUCTION FUNCTION STUDIES IN HIGHER EDUCATION	9
IV. PRODUCTION FUNCTION FOR FOUR YEAR HISTORICALLY BLACK COLLEGES	14
V. CONCLUSIONS	21
VI. NOTES	37
VII. REFERENCES	42

List of Illustrations

	Page
Figure 1. - Price Efficiency for a Multi-Resource, Multi-Outcome Production Process	4
Figure 2. - Preference Efficiency for a Multi-resource, Multi-outcome Production Process	4
Figure 3. - Relationships Among Unit Cost, Regression and Frontier Analysis	10
Figure 4. - Graph of Linear and Cobb-Douglas Production Functions for Constant Capital: Public and Private Institutions	30
Figure 5. - Graph of Linear and Cobb-Douglas and CES Production Functions for Constant Capital: Public and Private Institutions Combined	31

LIST OF TABLES

	Page
Table 1 - Properties of Productions Used	8
Tables 2-5 Student-Faculty Ratios for Private Highly Selective Liberal Arts Colleges	12
Table 6 - Public and Private Black Colleges: General and Technical Efficiency in Realizing Public Policy Objectives 1972-1973	17
Table 7 - Linear Production Function Estimates	23
Table 8 - Cobb-Douglas Production Function Estimates	25
Table 9 - CES Production Function Estimates	27
Table 10 - Comparison of Characteristics of Production Functions	28
Table 11 - Specification Analysis of Three Production Functions	29
Table 12 - Definition of Variables	32
Table 13 - List of Data	33
Table 14 - Zero-Order Correlation Matrices	35

I. INTRODUCTION

For most of this decade many colleges and universities have claimed some degree of financial distress, mounted stringent expenditure stabilization programs, sought increased federal, state and philanthropic support, and worried about the imminent decline in youth demand for higher education.[1] Efforts to narrow the income-expenditure (expectations) gap have been modestly successful at achieving a "precarious balance", partially funded by the 9% growth in enrollments between 1974 and 1975. However, the small enrollment decline between 1975 and 1976, the first such decline since the early 1950's, and the projected 10% - 15% enrollment decline between 1980 and 1990 raise serious doubts about the viability of institutions making only marginal expenditure adjustments.[2] What is needed is a serious reevaluation of the use of resources by colleges and universities and how efficiently they employ resources to provide educational and research services.

Although cost studies of higher education abound, very little is known of the relative efficiency of resource use. In this paper we review the previous research on the productivity of higher education and estimate empirically the resource-use behavior of relatively efficient colleges. The focus of our study for purposes of illustration is four year historically black colleges, which have been of particular interest to both public and private funders.[3] Ours is not a study of the financing of black colleges[4] but illustrates how relatively efficient resource use can be identified and how alternative production functions can be used to describe the resource use characteristics of relatively efficient institutions.

This approach can be used to answer a number of important policy questions:

1. How much could an average institution save, or have available for alternative use, by altering its resource use patterns to match those of relatively efficient institutions (controlling for type, control, and other important characteristics of institutions)?
2. Are there economies of scale, implying that as colleges expand in size they could or should become more efficient?

3. Among relatively efficient institutions, what mix of inputs (labor, capital) is technically optimal (or what are the marginal productivities of labor and capital)?
4. What type of information is needed by external funding agencies to develop grant policies that will encourage institutional efficiency?

We proceed by discussing the essentials of the theory of production function analysis (section II), reviewing the previous resource use analyses in education (section III), describing our empirical study in terms of the data and the statistical results (section IV), and concluding with summary observations (section V).

II. THEORY OF PRODUCTION FUNCTION ANALYSIS

There are many different perspectives from which we can ask about the efficiency of resource use in higher education: the investments of time and money by students as they pursue their education[5]; the investment of money in education by governments and philanthropy in the hope of accomplishing socially desirable objectives[6]; the acquisition and use of resources by individual colleges as they seek to accomplish the college's objectives[7]; the impacts on student achievement, attitudes or other characteristics associated with institutional resource use[8]; or the interaction of faculty and student decisions about how to allocate their time and efforts[9]. In this paper we adopt the view of the college or university as an organization with many and often conflicting objectives, seeking to acquire and use resources in the most productive manner.

Concepts of Efficiency

The "theory of the firm" from neoclassical microeconomics is a useful point of departure for our discussion[10]. In essence, the theory of the firms assumes the existence of a production function, where a production function describes the maximum output that can be achieved with a given mix of inputs and, correspondingly, the minimum amount of inputs needed for a given level of output. A production function is the technically most efficient means by which "physical" units of inputs can be used to produce "physical" units of outputs, where "physical" refers to direct quantity measures and not economic values of inputs or outputs. Colleges or universities that make the most efficient use of resources relative to other institutions producing services of comparable quality are said to be "production efficient" because they would be on the production frontier.

There are two other concepts of efficiency from the neoclassical theory of the firm: "price or allocative efficiency" which describes the least costly mix of resource inputs and "preference efficiency" which describes the utility maximizing mix of multiple outputs[11] (see Figures 1 and 2). Leibenstein (1976) has extended these concepts by suggesting a fourth type of efficiency, which he labels "x-efficiency", describing managerial ability (and willingness) to enable organizations to accomplish their objectives.

Figure 1. Price Efficiency for a Multiresource, Multioutcome Production Process

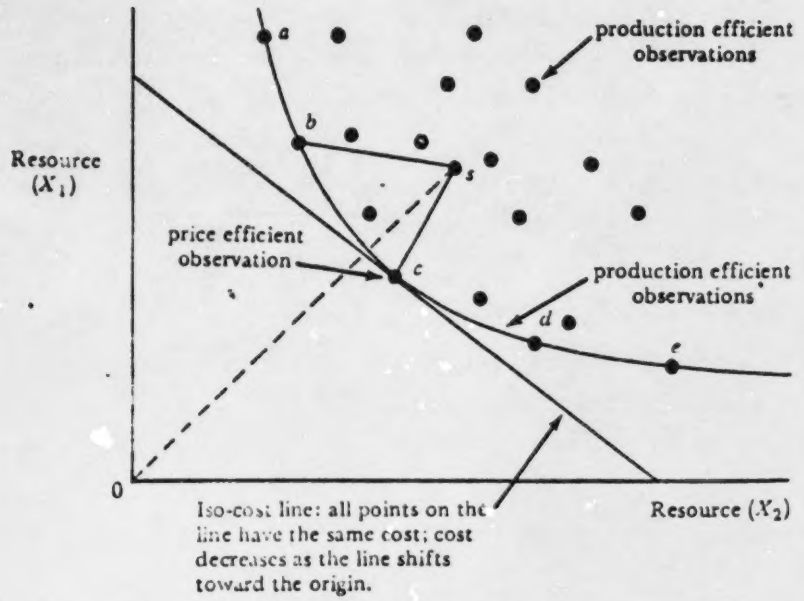
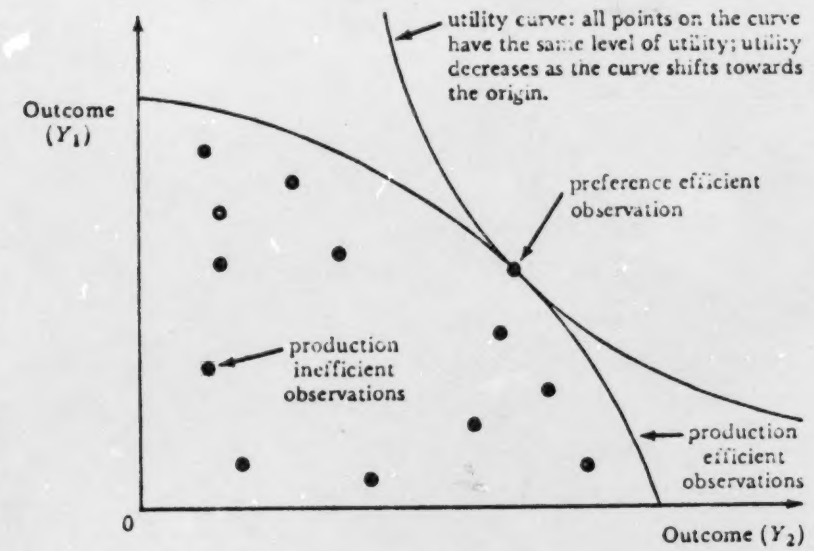


Figure 2. Preference Efficiency for a Multiresource, Multioutcome Production Process


utility curve: all points on the curve have the same level of utility; utility decreases as the curve shifts towards the origin.
Outcome (Y_1)
preference efficient observation
production inefficient observations
production efficient observations
0
Outcome (Y_2)

Source: Carlson (1975), pp. 42-43.

While most of the theory of the firm was developed for profit seeking organizations, very little of the formal analytic development of the theory is limited to for-profit organizations. It is often the case that the outputs to be maximized by a college or university are diverse, objectives are vague and unspecified, decision making is often diffuse and the production technology is often poorly understood.[12] This ambiguity is shared by profit seeking consulting firms, research laboratories, medical practices, and other service firms as well. Once the list of desirable outcomes is identified, there is an incentive for all organizations to be more production efficient--otherwise some desirable outcomes are not being produced or unnecessary inputs are being purchased. Different colleges clearly face different input prices and have different outcome preferences; consequently, the only generalizable conclusion about allocative and preference efficiency is that they lie on the production efficient surface, as shown in Figures 1 and 2. The remainder of our analysis focuses on the production function itself.

Limitations of Production Function Analysis

Examining the actual resource use behavior of colleges and universities and inferring the relative production function from observed data have several implicit assumptions which have been challenged by some researchers. These assumptions and their critiques follow:

1. All institutional managers have full information about the production function. Cohen and March (1974) and Levin (1971) dispute this assumption and claim the technology is largely unknown by educational decision makers at all levels. Leibenstein (1976) argues that even with perfect information, managers have differential ability and enthusiasm which make the assumption moot.

2. Managerial discretion exists over which inputs are obtained and how they are used to produce educational services. In reality, tenure, collective bargaining, long term debts and many other rigidities limit managerial discretion. The perceived degree of managerial flexibility is often very low, even when, in legal terms, it is substantial.

3. A competitive market exists for the services produced by colleges and universities. There is an assumption of nearly rapacious competition

in higher education, especially between public and private institutions,[13] but the available evidence of student choice[14] and institutional marketing[15] suggests that the current market is far from competitive. This is not to say that the traditional college-going market is not limited in magnitude. Doermann (1976) points out how few youth there are in high academic achievement, high income categories--or high academic achievement of any income.

While educational services are not be sold at prices reflecting a competitive market, almost all institutions, public as well as private, view their income as largely proportional to student enrollment (and perhaps research services provided). However, it is true that there are few market indicators of success such as sales, profits, losses, rates of return or shares of market (Levin 1971, Cohen and March 1974).

4. Market determined prices exist for all inputs and outputs and are known to all managers. This is a corollary to the third assumption and has received much of the same criticism. Violating this assumption is mainly a limitation to interpreting allocative and preference efficiency and this suggests another reason to expect many institutions to be inefficient when compared with production efficient institutions. Additional complications here are that (1) the federal and state financial aid programs effectively differentiate prices by the income and assets of parents or students rather than the quality or quantity of the educational service; and (2) multiple outcomes (such as research, graduate instruction and undergraduate instruction) are differentially valued by different funders.[16]

5. Colleges and universities are free from external constraints on their choices of resource use. In recent years the explosive growth of federal, state and local regulations, procedures, and requirements have placed many, complex restrictions on educational (and most other) organizations. While some attempts have been made to quantify the institutional costs of compliance with these procedures,[17] relatively little is known about how much these procedures affect production efficiency.

6. Inputs and outputs can be measured. This assumption is so simple it is rarely stated, but it is rarely true in education. First, it is difficult to find agreement on which inputs and outputs ought to be measured. Then it is difficult to find agreement on valid, unbiased, robust

measures of quantity and quality. Finally, we have little theory to guide the identification of exogenous and intervening variables necessary to describe fully the relationships among inputs and outputs. All empirical studies in educational productivity have used proxy measures for inputs and outputs. We do, too, and explain our choices in a following section.

Functional Forms of Production Functions

We considered three functional forms to describe the transformation of inputs into outputs along the production efficient frontier: (1) linear;[18] (2) Cobb-Douglas; and (3) Constant Elasticity of Substitution (CES). For descriptive purposes, we define:

Q_i = quantity of output of institution i in a given year. (We assume that units of input and output are qualitatively similar within and among institutions.)

L_i = quantity of labor input in institution i

K_i = quantity of capital input in institution i

u_i = random error terms [$iid(0, \sigma_u^2)$].

The linear model is

$$Q_i = a_o + a_1 L_i + a_2 K_i + u_i \qquad (1)$$

where a_u, a_1, and a_2 are constants.

The Cobb-Douglas[19] Model is

$$Q_i = A L_i^{\alpha} K_i^{\beta} \varepsilon^{u_i} \qquad (2)$$

where A, α, and β are constants.

The CES[20] Model is

$$Q_i = \gamma[\delta K_i^{-\rho} + (1-\delta) L_i^{-\rho}]^{-\nu/\rho} \varepsilon^{u_i} \qquad (3)$$

where γ, δ, ρ, and ν are constant. The efficiency parameter, γ, describes the overall efficiency of the technology while δ describes the capital intensiveness of the technology of production.

The properties of these three functional forms are summarized in Table 1. The functional transformation and statistical estimation procedures will be discussed in a later section.

Table 1

Properties of Production Functions Used

	Linear	Cobb-Douglas	CES
Marginal Product of Labor	a_1	$\alpha\frac{Q}{L}$	$(1-\delta)\nu Y^{-\rho/\nu} Q^{1+(\rho/\nu)} L^{-\rho-1}$
Marginal Product of Capital	a_2	$\beta\frac{Q}{K}$	$\delta\nu Y^{-\rho/\nu} Q^{1+(\rho/\nu)} K^{-\rho-1}$
Returns to Scale	1.0	$\alpha+\beta$	ν
Elasticity of Substitution (σ)	∞	1.0	$\frac{1}{1+\rho}$
Marginal Rate of Substitution of Labor for Capital	$\frac{a_2}{a_1}$	$\frac{\beta}{\alpha}\frac{L}{K}$	$\frac{\delta}{1-\delta}\left(\frac{L}{K}\right)^{(1+\rho)}$

Source: Brown (1966)

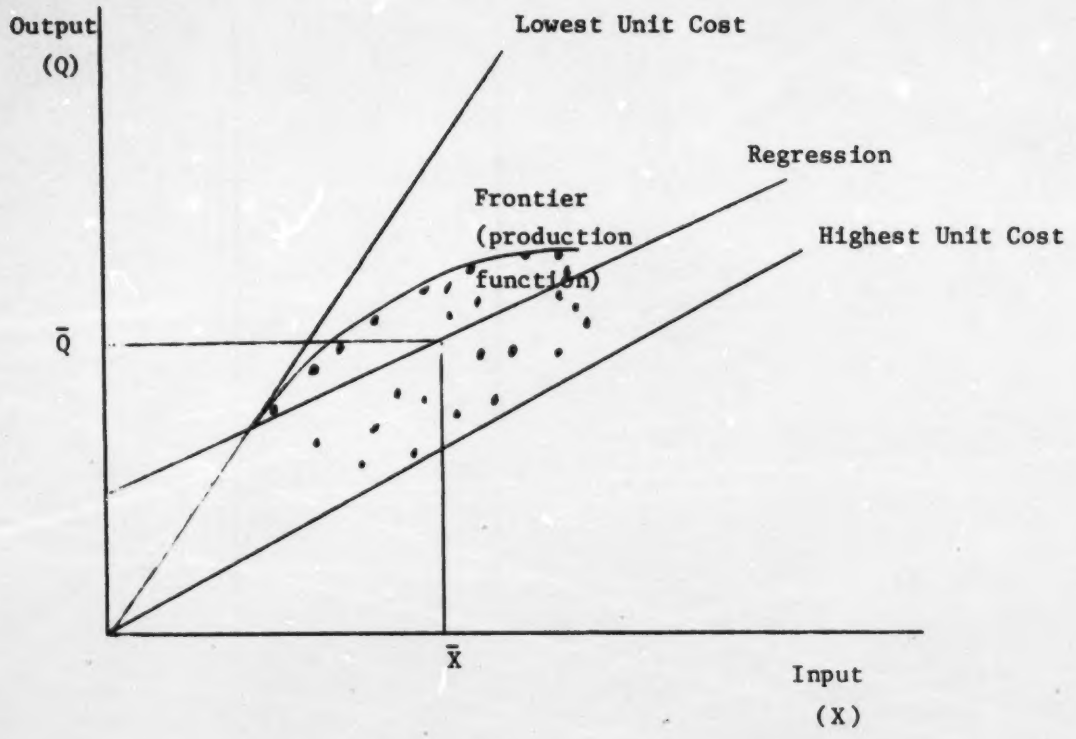
III. PREVIOUS PRODUCTION FUNCTION STUDIES IN HIGHER EDUCATION

One key concept differentiating production function studies from other studies of resource use in higher education is that a production function describes the most efficient use of resources actually observed in institutions, not the average, median or any other measure of resource use. In a recent review of production and cost behavior in higher education, Carlson (1976) classifies previous studies into (a) frontier studies, which attempt to identify those institutions or observations which are on the production function, and studies of average relationships by (b) descriptive statistics, (c) regression analysis, and (c) synthetic analysis, all of which attempt to describe average cost behavior by unit costs, linear and non-linear regression, and the economic-engineering paradigm that assumes an explicit production function as a basis for analysis.[21]

Average Resource Use Studies Called Production Functions

Although the maximizing concept underlying a production function has a relatively long history, many recent studies have used a total sample, not just those observations on the frontier, as the basis for estimating equations loosely called production functions.[22] Examples of regression studies of "production functions" are: Radner and Miller's (1970) linear activity analysis of student-faculty input-output relationships; Breneman's (1970) linear, departmental model of doctoral degree production; Blau's (1972) linear-logarithmic model of administrator-faculty ratios; Southwick's (1967) linear, activity analysis of various types of staff and capital needed for undergraduate, graduate, and research outputs for 146 universities; Freire and daSilva's (1975) Cobb-Douglas form regression of enrolled students as a function of the number of professors and teaching assistants in schools of law and art in five Portuguese universities (1959-1971) and Verry and Davies' (1976) linear and logarithmic analysis of enrolled students as functions of departmental salaries, faculty teaching and research time, and a number of other factors. All of the regression studies use ordinary least squares to estimate the unknown parameters. Consequently, all of these studies describe average resource use rather than efficient resource use and could be very misleading if assumed to reflect efficient resource use. Figure 3 shows the relationships among unit cost analysis, regression analysis and frontier analysis of a simple production function.

Figure 3. Relationships Among Unit Cost, Regression and Frontier Analysis



Frontier Production Function Studies

Studies of the frontier production function conceptually involve two steps: (a) identifying the institutions or observations that are on the frontier, that is, those that are efficient relative to other institutions producing comparable quality outcomes; and (b) estimating a continuous production function through the frontier observations. To our knowledge, this is the first study to complete both steps.

There are several methods to identify frontier observations including constrained residual regression, quadratic programming and linear programming.[23] The most general and least sensitive to prior specification of the functional form of the production function is the linear programming technique. Essentially, the linear programming approach to identifying frontier observations requires the researcher to identify the input, output and characteristic (intervening) variables of the institutions in the total sample, where all institutions are judged to have comparable quality outcomes or where one or more measures of quality are explicitly included as characteristic variables. Linear programming is then used to maximize the attainment of outcomes given the constraint of the available input resources.[24] The result of this analysis applied to each institution is a production efficiency index that will be 1.0 for all institutions or observations on the frontier and less than 1.0 for all other institutions or observations.[25] Thus all observations on the frontier can be identified, which is the first step in the estimation of the production function.

Examples of the application of this linear programming approach to identifying frontier observations include Levin's (1974) study of student achievement in which he found substantial sensitivity to extreme data points and very little relationship between the frontier and the regression surfaces. Carlson's (1975) study of private, highly selective liberal arts colleges and public, comprehensive colleges and universities shows the significant resource use differences between average and frontier institutions (see Tables 2, 3, 4, 5). This preliminary analysis suggests that the differences in relative production efficiency among institutions is substantial and institutions as well as funders might benefit greatly from a careful analysis of resource use.

Table 2 Student-Faculty Ratios for Private Highly Selective Liberal Arts Colleges

		Total Enrollment/Faculty
Average		13.8
Frontier:	typical institution	18.7
	high level of research	17.5
	large number of fields	16.5

Table 3 Cost-Per-Student Ratios for Private Highly Selective Liberal Arts Colleges

		Cost/Total Enrollment
Average		$1,371
Frontier:	typical institution	768
	low level of part-time undergraduates	796
	high level of part-time undergraduates	719
	small institution	780

Table 4 Student-Faculty Ratios for Public Comprehensive Colleges and Universities

		Total Enrollment/Faculty
Average		23.5
Frontier:	typical institution	32.1
	no teaching assistants	29.8
	large number of fields	31.6
	low level of part-time undergraduates	29.2
	high level of part-time undergraduates	33.6

Table 5 Cost-Per-Student Ratios for Public Comprehensive Colleges and Universities

		Cost/Total Enrollment
Average		$883
Frontier:	typical institution	514
	high level of graduate students	525
	no research	499
	high level of research	533
	small institution	515

Source: Carlson (1975) p. 50

While the linear programming approach is useful to identify frontier observations, it is a very awkward formulation to describe the frontier (or efficient surface). By constraining all but two variables, one can use the linear programming solution to trace out isoquants and frontier production relationships (Carlson, 1976). Alternatively, one can compare efficient and average institutions (Levin 1974, Carlson 1975). In this study we go to the next step of estimating continuous functions for the frontier institutions.

IV. PRODUCTION FUNCTION FOR FOUR YEAR HISTORICALLY BLACK COLLEGES

The recent and growing federal interest in financially supporting most historically black colleges,[26] the major philanthropic interest in their financial and academic well being,[27] the Adams vs. Richardson ruling that all states must prepare and submit to HEW plans for integrating the predominantly black public colleges,[28] and the general concern about their long-term financial viability encouraged us to pay particular attention to the four year historically black colleges, both public and private. The Lupton et al. (1976) report on indicators of financial health showed black colleges apparently to be in more precarious financial condition than many other colleges and, yet, their history and their contribution is too great[29] for public policy to allow them to wither. We clearly need to understand more about the efficiency of resource use in all colleges and particularly in financially hard-pressed historically black colleges.

Data

We have used the college as our unit of analysis, unlike many private sector production studies which use two or three digit industries. Data submitted by institutions to the Higher Education General Information Survey (HEGIS) and to the Developing Institutions Program in application for funding were available.[30] Both of these sources were incomplete and, at times, inconsistent. Supplementary sources were used including the AAUP Bulletin, Yearbook on Higher Education, American Universities and Colleges, and various NCES publications. Some data for academic years 1972, 1973, and 1974 were available but the most complete data set was 1972. Consequently, all the statistical results are quoted for academic year 1972-1973.

Outputs

Conceptually, in the area of instruction, a college makes certain educational services available to students. In turn, students bring a variety of preparations, motivations, aspirations, and abilities to their learning. The net effect of all these interactions is, hopefully, some change in student achievement or attitudes. There are at least two schools of thought on how to conceptualize the outcomes of a college: (a) the change in student characteristics associated with various institutional characteristics;[31]

or (b) the characteristics of learning opportunities (or services) made available by a college.[32] We argue that the changes of student characteristics reflect their learning function and are only indirectly related to a college's production function (just as one firm's production function is not linked at all to another firm's actions but the allocative efficient mix of inputs reflects market prices which are (implicitly) affected by another firm's (or industry's) supply function).

The outputs appropriately attributed to a college are the magnitude, quality and duration of services made available. We generally have no data on these and, as usual, only observe the intersection of institutional supply and student demand.[33] Given that student admission and faculty hiring decisions are made up to a year in advance and that in 1972 the black colleges were in a relatively slow rate of change, we assume that student enrollment is an adequate proxy for the magnitude of instructional services made available. Also, at this level of analysis we assume that the relative quality of services offered by our sample colleges is comparable. This is not to say an equal "quality" student attends each college and we recognize glaring exceptions to this assumption, but we know of no measures of the quality of institutional services.

Graduate, research and public service outcomes are obviously of interest to some and, in terms of service, probably all colleges.[34] With a very few exceptions, the colleges we studied have very small graduate programs and we alternately used undergraduate and total enrollment as an outcome measure.[35] We have no data on public service outcomes or expenditures and that variable is omitted.

Labor Inputs

Two labor variables were used consistently: the full-time equivalent of instructional staff, including teaching assistants through full professors teaching in degree and non-degree, resident and extension programs; and full-time administrators with "professional academic rank or status." For total labor we used the sum of these two variables, even though the number of non-instructional, non-administrative support staff was not available in our data base.[36] In some analyses we disaggregated instructional staff by highest degree attained, but high multicollinearity

prevented statistically significant parameter estimates. These results will be briefly discussed in a following section.

Capital Input

Ideally, we sought a measure of the annual cost of capital used by colleges but the only capital measure available consistently is the capital stock of colleges: the dollar value of land, buildings, and equipment.[37] We assume implicitly that the capital services and their cost are proportional to the value of the capital stock.

These variables for inputs and outputs are imperfect descriptors of educational resources used and services provided, but they are a starting point. Every empirical study of educational production has been limited by incomplete, inadequate or inaccurate data, as have similar studies of private sector production,[38] but most of these empirical studies of educational production have been conceptually limited to average production behavior. We hope our conceptually more consistent approach to production function identification and estimation will add urgency to the necessity for more meaningful data collection.

Identification of Production Frontier

Rodgers (1976) applied Carlson's (1975) linear programming technique to the data for 98 historically black colleges. Two different outcome measures were used: total enrollment, which Rodgers called "general efficiency;" and low income and ethnic minority enrollment, which he called "technical efficiency." Out of the 98 original institutions, 44 were identified as efficient by both measures. (See Table 6.) Two of these colleges were later excluded because they were so atypical of the remaining 42 which then constituted the sample for the production function estimation.

Estimation of Production Functions for Efficient Institutions

The results of estimating by ordinary least squares the linear production function, the log-log transformation of the Cobb-Douglas production function and Kmenta's (1967) log-log approximation of the CES production function are shown in Tables 7, 8, and 9 respectively. As shown in these tables, most coefficients are significant for the two input variables,

TABLE 6

General and Technical Efficiency in Realizing Public Policy Objectives 1972-73

Institutional Number	PUBLIC BLACK COLLEGES			PRIVATE BLACK COLLEGES		
	General Efficiency Ranking	Technical Efficiency Ranking Access Black Students	Low-Income Students	General Efficiency Ranking	Technical Efficiency Ranking Access Black Students	Low-Income Students
1	1.00	1.00	1.00	1.00	1.00	1.00
2	1.00	1.00	1.00	1.00	1.00	1.00
3	.848	NA	NA	.716	.816	.861
4	1.00	1.00	1.00	.254	1.00	.919
5	1.00	1.00	1.00	1.00	1.00	1.00
6	.848	.826	.403	1.00	1.00	1.00
7	1.00	.745	.929	1.00	1.00	1.00
8	1.00	1.00	1.00	.936	1.00	1.00
9	1.00	1.00	1.00	1.00	.623	.579
10	1.00	1.00	NA	1.00	1.00	1.00
11	1.00	.512	NA	.707	.550	.644
12	1.00	.723	1.00	1.00	.702	.288
13	1.00	1.00	1.00	.892	.766	.750
14	.934	.595	1.00	1.00	1.00	1.00
15	1.00	1.00	1.00	1.00	1.00	.981
16	1.00	.917	.736	1.00	.932	.019
17	1.00	1.00	1.00	.893	1.00	1.00
18	.372	.691	1.00	1.00	1.00	1.00
19	.948	.771	.688	.867	.917	.656
20	1.00	1.00	NA	.662	1.00	1.00
21	1.00	.555	.313			
22	1.00	.837	.537			
23	1.00	1.00	1.00			
24	1.00	1.00	1.00			
25	.607	.827	.722			
26	1.00	1.00	1.00			
27	1.00	1.00	1.00			

Source: Rodgers (1976)

labor and capital; form and disaggregation of labor and other expenditures adds little information. Most of the functions explain 80% to 90% of the variation in output or log (output), which is very significant, especially for a cross-sectional study.

The linear production function estimates (Table 7) are easily interpretable. For public institutions, labor productivity ranges between 14.5 and 16.0 students per faculty member, while for private institutions, the labor productivity ranges between 4.3 and 7.8 students per faculty member. Recall that both sets are relatively efficient institutions. When the faculty are disaggregated by highest degree achieved, the results become confounded. For both public and private colleges, the M.A. faculty constitute approximately two-thirds of the total faculty (see Table 13) and are correlated about 0.9 with the Ph.D. faculty and 0.5 with the B.A. faculty (see Table 14). Therefore, it is not surprising that only the M.A. faculty enter the linear production function with significant coefficients for public and all institutions, though not for private institutions.

The capital productivity is never significantly different from zero for public institutions or all institutions. The only significant coefficient for capital is for private colleges, 3.4 students per $100,000 in capital investment. This is the high end of the range of coefficients estimated, the remainder of which were all closer to zero.

The Cobb-Douglas production function estimates (Table 8) are a little less directly interpretable because all the dependent and independent variables are expressed in logarithms. However, a pattern similar to the linear production function emerges. Aggregate labor variables are generally significant and their coefficients are much larger in magnitude than capital variables. One difference is that Ph.D. faculty rather than M.A. faculty are significant for public and all institutions. For private institutions, annual educational and general expenditures other than instruction are also significant.

The returns to scale are indicated by the sum of coefficients column in Table 8. In general, the sums of coefficients do not significantly differ from 1, indicating constant returns to scale. However, for all of the equations for public institutions the sum of coefficients range

from 1.07 to 1.13 while the corresponding figures for private institutions range from .87 to .93. This symmetric and opposite variation about 1.0 suggests that the returns to scale of public institutions are marginally greater than for private institutions, when comparing institutions on the efficient surface.

The CES production function estimates (Table 9) are even less directly interpretable than the Cobb-Douglas results, but do shed some insight. The measure of technology, which is the constant term in the log-log form for estimation, is consistent between the C-D and CES forms for public institutions but is different by a factor of 3 or 4 between the C-D and CES forms for private institutions. It is unclear why private institutions should exhibit higher structural productivity. Once again the labor coefficients are all significant and the capital coefficients are insignificant and even have negative signs.

To provide some comparison of the characteristics of the three different forms of production functions, Table 10 reports the marginal products of capital and labor, the returns to scale, the elasticity of substitution, and the marginal rate of substitution of labor for capital. Where there are corresponding characteristics their magnitudes are quite comparable. Substituting the mean values of variables from Table 13, for example, the C-D marginal product of labor for public institutions is 13.7 to 15.0 students per faculty which compares to the 14.5 to 16.0 range for the linear form. The returns to scale of public institutions is 1.13 for the CES and 1.09-1.13 for the C-D. The MRS of labor for capital for public institutions is .0012 to .0021 for the C-D. Similar comparative results obtain for private institutions. Again using mean values the marginal products of labor overlap, 4.3 to 7.8 for linear, 5.8-7.7 for C-D. The marginal products for capital are .034 for linear and .036 for C-D. The returns to scale are .866-.928 for C-D and .897 for CES. The MRS of labor for capital is .0044-.0047 for the C-D, about twice the MRS for public institutions.

While Table 10 provides a comparison of the characteristics of the three production functions estimated, it does not answer the question: Which functional form best describes the efficient surface identified by linear programming? To resolve this question, Table 11 presents a

specification analysis of the three functional forms: linear, Cobb-Douglas and CES. Observe that all the estimated equations are significant at the 99% confidence level. However, the linear and C-D functional forms are substantially better than the CES. The best functional form ("best" in terms of minimal sum of squared linear deviations) describes 89% to 91% of the variation in enrollment spaces produced by institutions determined to be on the efficient surface. These results are shown graphically in Figures 4 and 5.

V. CONCLUSIONS

There are two important research contributions of this paper: (1) it is possible to identify relatively efficient colleges and universities and to describe their resource use characteristics by mathematical forms traditionally used to describe production functions of profit-seeking firms; and (2) all of the mathematical forms we considered have similar quantitative results for important resource use characteristics. For four year, historically black colleges, the sample of institutions analyzed by this report, we can now describe the resource use behavior of relatively efficient institutions.

At least for the variables considered, there are significant economies achievable by adopting the resource use patterns of relatively efficient institutions. Rodgers' (1976) work with four year, historically black colleges confirms Carlson's (1975) results of substantial resource savings in moving from average to efficient resource use (see Tables 2, 3, 4 and 5).

While public, four year, black colleges exhibit greater returns to scale than do private, four year, black colleges, neither group exhibits statistically significant economies of scale. In other words, for enrollments between 456 and 5678, and faculty between 43 and 295, once an institution is on the efficient surface there are no demonstrable economies of growth. The frequent assertion that larger institutions are more efficient could still be true if one separates movement towards the efficient surface (which is not a function of growth but of more efficient resource use) from movement along the efficient surface (which has no apparent economies of scale). Colleges and universities should strive to be efficient at their current size and expand or contract only for programmatic or market demand reasons, not in the expectation of greater efficiency.

Efficient, private, four year, historically black colleges have a larger capital component than do corresponding public colleges. At about one-third the average enrollment of public, four year, historically black colleges, the corresponding private colleges average one-half the average capital investment of the publics. The marginal product of capital is not significant for public colleges but is significant for private colleges. This suggests that public, four year, historically black

colleges may be under-capitalized. There is certainly the reality that the public marginal product of labor is two or three times that for private institutions.

The policy implications of this analysis for institutional funders are manifold. Funders should encourage institutions to adopt more efficient resource use patterns. The substantial resource gains from greater efficiency, on the order of 30-50%, are far larger than virtually all federal and philanthropic funding programs. Funders should not encourage institutions to expand in the quest for efficiency. Market demand, manpower demand and constant returns to scale all argue against expansion. The large differences in labor and capital productivity in the public and private sectors should be examined in detail; both sectors can learn from each other.

As usual, this is just a beginning of the research needed to understand the resource use patterns of institutions of higher education. Hopefully, it is a fruitful beginning.

Table 7

Linear Production Function Estimates

Dependent Variable: Total FTE Enrollment, 1972
(Standard Errors in Parentheses)

	Constant	PhD	MA	BA	Faculty	Admin	Labor	Capital	Other Expen.	R^2 a/
PUBLIC (N=20)										
1.	88.5168						15.537*	-0.002		.90
							(1.666)	(.018)		
2.	44.3318						14.532*	-0.011	0.093	.90
							(1.939)	(.020)	(.092)	
3.	137.9821				15.966*	10.859		-0.004		.89
					(1.895)	(9.217)		(.019)		
4.	74.5118				14.839*	12.056		-0.012	0.086	.89
					(2.298)	(9.382)		(.021)	(.098)	
5.	580.8317	16.019	17.727*	14.065		-3.588		-0.012		.85
		(16.941)	(6.119)	(31.235)		(12.413)		(.034)		
6.	555.0985	18.286	16.623*	2.370		-2.713		-0.025	0.068	.85
		(18.248)	(6.816)	(42.274)		(12.956)		(.045)	(.159)	
PRIVATE (N=22)										
7.	220.8950						6.786*	0.034*		.88
							(1.810)	(.013)		
8.	212.5886						4.276*	0.013	0.143*	.90
							(2.103)	(.016)	(.072)	
9.	234.9827				7.811*	4.365		0.028		.88
					(2.311)	(3.796)		(.016)		
10.	226.0203				5.268*	1.999		0.007	0.142	.90
					(2.514)	(3.733)		(.018)	(.073)	
11.	220.5636	7.149	6.891	15.523		5.263		0.034		.87
		(6.508)	(4.499)	(14.695)		(4.113)		(.017)		
12.	205.4100	3.817	5.495	9.569		3.046		0.012	0.133	.88
		(6.532)	(4.372)	(14.474)		(4.150)		(.012)	(.082)	

--continued--

Table 7, Linear Production Function Estimates, continued.

	Constant	PhD	MA	BA	Faculty	Admin	Labor	Capital	Other Expen.	R^2 a/
PUBLIC AND PRIVATE (N=42)										
13.	-208.3361						15.838* (.809)			.90
14.	-162.5835						16.231* (1.513)	0.009 (.016)	-0.060 (.065)	.90
15.	-80.4973				16.919* (1.528)	6.028 (5.648)		-0.007 (.014)		.91
16.	-27.0282				17.475* (1.621)	6.320 (5.651)		0.002 (.016)	-0.064 (.063)	.91
17.	16.6438	12.523 (10.233)	18.890* (3.854)	17.468 (20.267)		1.263 (6.753)		0.003 (.020)		.87
18.	98.5421	11.877 (10.117)	19.317* (3.819)	33.689 (23.205)		2.164 (6.701)		0.022 (.024)	-0.115 (.083)	.88

*Coefficient significant at the 0.05 level.

a/Adjusted R^2. Since the sample size is small, these values represent a more conservative estimate of the percent of variance explained.

Table 8

Cobb-Douglas Production Function Estimates

Dependent Variable: Log of Total FTE Enrollment, 1972

(Standard Errors in Parentheses)

	CONSTANT	PHD	MA	BA	FACULTY	ADMIN	LABOR	CAPITAL	OTHER EXPEND	SUM OF COEFF	R^2 b/
PUBLIC (N=20)											
1.	1.2800						.951* (.133)	.177 (.147)		1.128	.86
2.	1.1742						.914* (.162)	.155 (.159)	.063 (.146)	1.132	.86
3.	2.0001				.922* (.137)	.069 (.039)		.111 (.151)		1.102	.88
4.	1.7890				.866* (.169)	.072 (.040)		.089 (.159)	.084 (.141)	1.111	.87
5.	3.6018	.803* (.278)	-.072 (.092)	.112 (.081)		.116 (.070)		.113 (.282)		1.072	.77
6.	3.0987	.746* (.294)	-.077 (.094)	.086 (.091)		.119 (.072)		.065 (.295)	.150 (.212)	1.089	.76
PRIVATE (N=22)											
7.	1.7621						.640* (.175)	.263* (.121)		.903	.75
8.	1.6759						.490* (.143)		.394* (.096)	.884	.84
9.	1.9721				.636* (.178)	.049 (.052)		.243 (.125)		.928	.76
10.	1.8326				.982* (.156)	.053 (.043)		-.021 (.135)	.398* (.131)		.83
11.	2.8663	.373 (.187)	.162 (.243)	.036 (.064)		.047 (.054)		.248 (.127)		.866	.75
12.	2.3228	.210 (.180)	.223 (.217)	.009 (.058)		.059 (.048)		.018 (.149)	.360* (.154)	.879	.80

Table 8 - Cobb-Douglas Production Function Estimates, continued. . .

	CONSTANT	PHD	MA	BA	FACULTY	ADMIN	LABOR	CAPITAL	OTHER EXPEND	SUM OF COEFF	R^2 b/
PUBLIC AND PRIVATE (N=42)											
13.	.7789						.884* (.112)	.258* (.092)		1.142	.88
14.	.5984						.849* (.118)	.213* (.104)	.094 (.102)	1.156	.88
15.	1.3727				.867* (.107)	.056 (.033)		.202* (.093)		1.125	.89
16.	1.1287				.829* (.111)	.057 (.033)		.149 (.103)	.113 (.094)	1.148	.90
17.	2.2763	.588* (.133)	a/	.089 (.099)		.063 (.040)		.311* (.116)		1.051	.85
18.	2.1323	.574* (.139)	a/	.084 (.051)		.063 (.040)		.291* (.128)	.047 (.122)	1.059	.85

*Coefficient significant at the 0.05 level

a/Variable does not enter equation with F=.01 or tolerance .001

b/Adjusted R^2. Since the sample size is small, these values represent a more conservative estimate of the percent of variance explained.

Table 9

CES Production Function Estimates

Dependent Variable: Log of Total FTE Enrollment, 1972

$$\text{LogQ} = \log \gamma + \nu\delta \log\left(\frac{K}{L}\right) + \nu \log L + \beta\left[\log\left(\frac{K}{L}\right)\right]^2$$

	Constant γ	$\nu\delta$	ν	β	R^2 b/
1. PUBLIC (N=20)	1.663	a/	1.130* (.111)	.020 (.016)	.87
2. PRIVATE (N=22)	7.047	-2.274 (1.545)	.897* (.114)	.303 (.184)	.77
3. PUBLIC AND PRIVATE (N=42)	3.334	-.965 (1.242)	1.152* (.067)	.142 (.144)	.88

*Coefficient significant at the 0.05 level

a/ Variable does not enter equation with F = .01 or tolerance .001.

b/ Adjusted R^2. Since the sample size is small, these values represent a more conservative estimate of the percent of variance explained.

Table 10

Comparison of Characteristics of Production Functions

Property	Linear		Cobb-Douglas		CES	
	Eq. No.	Range	Eq. No.	Range	Eq. No.	
PUBLIC(N=20)						
Marginal Product of Labor	(7.1-4)	14.5-16.0*	(8.1-4)	(.866-.951)*$\frac{Q}{L}$	(9.1)	$\frac{1.130Q}{L}$
Marginal Product of Capital	(7.1-6)	-.025--.002	(8.1-6)	(.065-.177)$\frac{Q}{K}$		0.0
Returns to Scale			(8.1-6)	1.089-1.132		1.130
Elasticity of Substitution		∞		1.000		0.0
MRS Labor for Capital			(8.1-4)	(.103-.186)$\frac{L}{K}$		0.0
PRIVATE(N=22)	Eq. No.	Range	Eq. No.	Range	Eq. No.	
Marginal Product of Labor	(7.7-10)	4.3-7.8*	(8.7-10)	(.482-640)*$\frac{Q}{L}$	(9.2)	$5.717Q^{.916}{}_{L}{}^{-.925}$
Marginal Product of Capital	(7.7)	.034*	(8.7)	.263*$\frac{Q}{K}$		$-4.100Q^{.916}{}_{K}{}^{-.925}$
Returns to Scale			(8.7-12)	.866-.928		.897
Elasticity of Substitution		∞		1.000		1.081
MRS Labor for Capital			(8.7, 8.9)	(.382-.411)$\frac{L}{K}$		$-.717(\frac{L}{K})^{.925}$
PUBLIC AND PRIVATE(N=42)	Eq. No.	Range	Eq. No.	Range	Eq. No.	
Marginal Product of Labor	(7.13-16)	15.8-17.5*	(8.13-16)	(.829-.884)*$\frac{Q}{L}$	(9.3)	$3.365Q^{.861}{}_{L}{}^{-.840}$
Marginal Product of Capital	(7.13-18)	-.007-.022	(8.13-15, 17, 18)	(.202-.311)*$\frac{Q}{K}$		$-1.533Q^{.861}{}_{K}{}^{-.840}$
Returns to Scale			(8.13-18)	1.051-1.156		1.153
Elasticity of Substitution		∞		1.000		1.190
MRS Labor for Capital			(8.13-15)	(.233-.292)*$\frac{L}{K}$		$-.456(\frac{L}{K})^{.840}$

*All coefficients in range reported are significant at the 0.05 level.

Table 11

Specification Analysis of Three Production Functions

(All transformed to linear before calculation)

	LINEAR	C-D	CES
PUBLIC (N=20)			
SSE	$3.75X10^6$	$5.02X10^6$	$2.59X10^7$
R^2	.91	.88	.36
$F_{2,17}$	85.944*	62.333*	9.5625* $F_{1,17}$
PRIVATE (N=22)			
SSE	$9.05X10^5$	$9.70X10^5$	$3.31X10^7$
R^2	.894	.886	NA
$F_{2,19}$	80.123*	73.833*	NA
PUBLIC AND PRIVATE (N=42)			
SSE	$8.08X10^6$	$1.03X10^7$	$1.20X10^6$
R^2	.905	.879	NA
$F_{2,39}$	185.968*	141.657*	NA

*Significant at the .01 level

Figure 4 - Graph of Linear and Cobb-Douglas Production Functions for Constant Capital: Public and Private Institutions

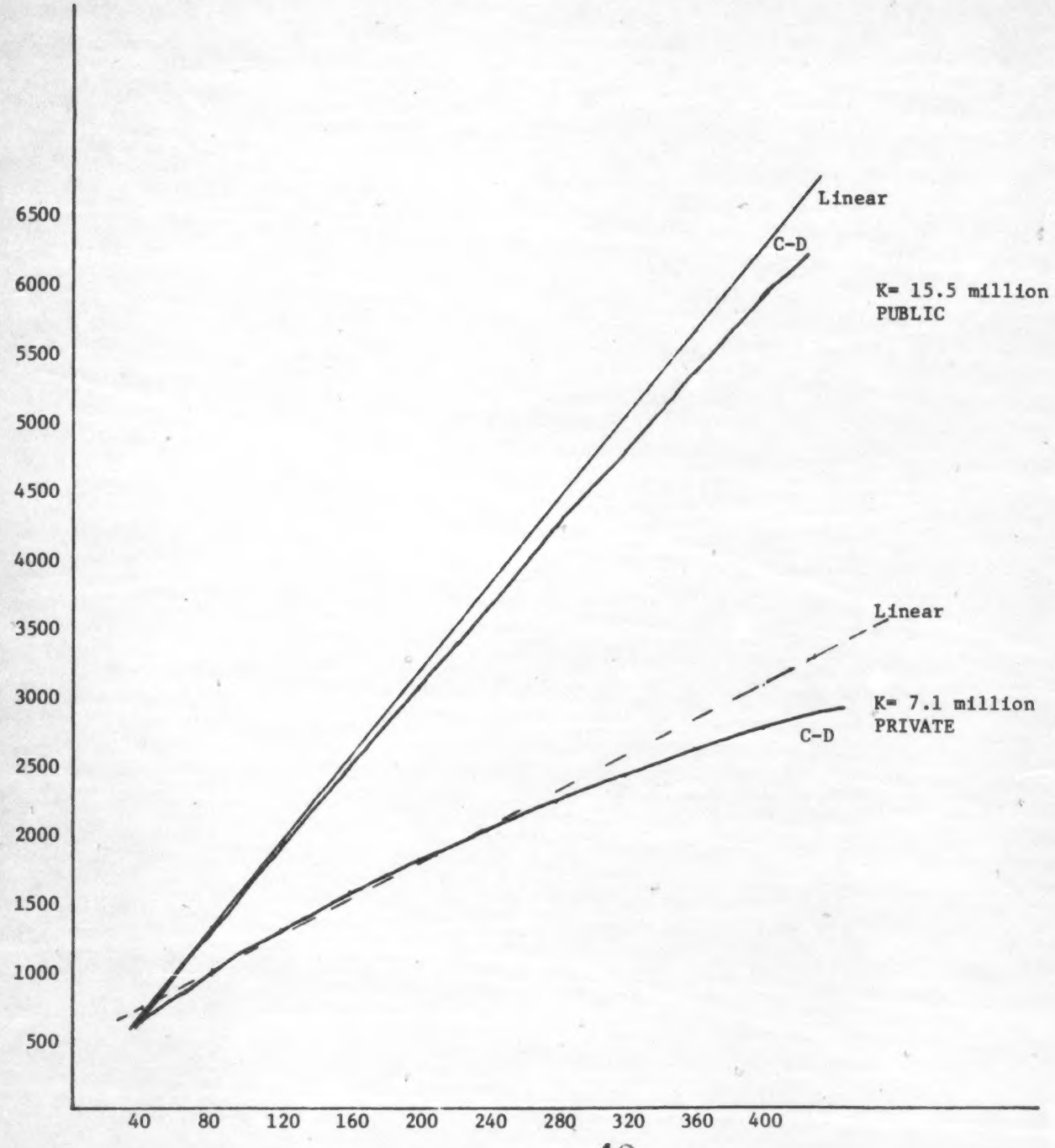


Figure 5 - Graph of Linear and Cobb-Douglas Production Functions for Constant Capital: Public and Private Institutions Combined

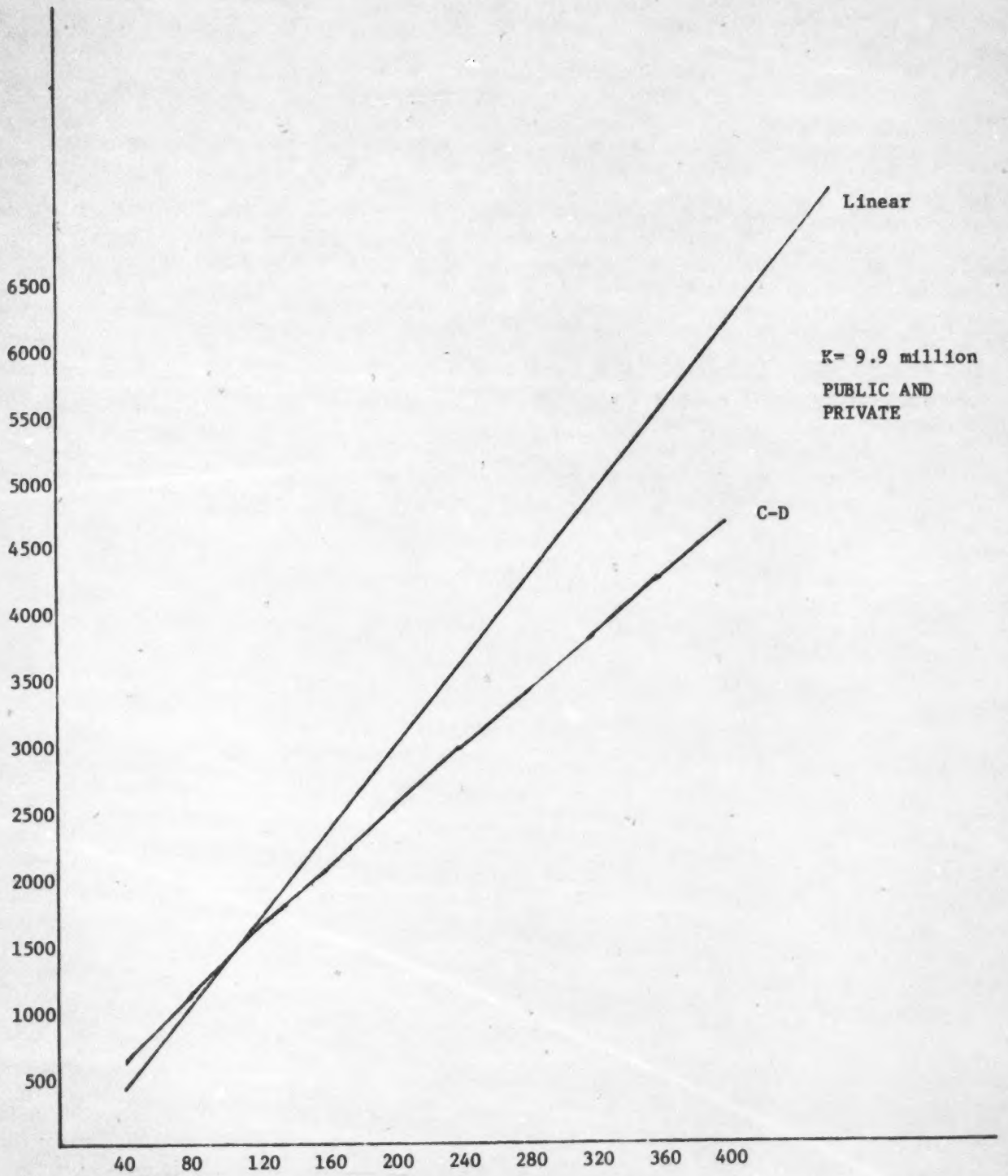


Table 12

Definition of Variables

Institutional Input Variables

PHD = The total number of persons working full-time* on the instructional staff whose highest earned degree is a doctoral degree.

MA = The total number of persons working full-time on the instructional staff whose highest earned degree is a master's degree.

BA = The total number of persons working full-time on the instructional staff whose highest earned degree is a bachelor's degree.

FACULTY = The total full-time equivalent (FTE) instructional staff. The FTE of part-time faculty is calculated by dividing the total number of hours worked by part-time faculty by the number of hours of normal full-time instructional load.

ADMIN = The total number of full-time individuals with professional rank serving as administrators.

LABOR = The total FTE faculty plus the total number of full-time administrators.

CAPITAL = The gross value of each institution's land, buildings and equipment fund as of the end of the preceding academic year.

OTHER EXPEN = The total current institutional expenditures less current expenditures for instruction.

Institutional Output Variables

TOTAL FTE ENROLLMENT = The total full-time equivalent enrollment. All students enrolled in credits equal to at least 75% of the normal full-time load, including students working on advanced and undergraduate degrees and in non-degree programs.

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*Full-time is defined as a staff member who devotes more than one-half of his or her time to an activity. The instructional staff includes degree and non-degree credit, resident and extension, senior and junior staff.

THE DATA

The following tables give the data used in the study and their means, standard deviations, and zero-order correlation matrices. The sources of the data are described in Section IV.

Table 13

List of Data

PUBLIC INSTITUTIONS

Case	Capital	Admin	BA	MA	PHD	FTE Faculty	FTE Enrollm	Labor	Other Expend a/
1.	18362.	19.	3.	108.	50.	169.	3009.	188.	3164.
2.	16091.	23.	0.	66.	43.	109.	2704.	132.	3654.
3.	10422.	30.	1.	88.	24.	115.	1926.	145.	2657.
4.	13908.	12.	3.	84.	47.	134.	2373.	146.	3341.
5.	24737.	41.	0.	167.	86.	254.	3913.	295.	4206.
6.	7745.	39.	13.	75.	35.	137.	2353.	176.	3442.
7.	23331.	b/	17.	190.	95.	347.	5312.	347.	7978.
8.	6500.	20.	0.	44.	20.	64.	771.	84.	1496.
9.	11135.	11.	9.	76.	29.	115.	2677.	126.	4897.
10.	14963.	30.	17.	200.	76.	295.	5058.	325.	5515.
11.	13259.	14.	5.	72.	44.	158.	2620.	172.	1656.
12.	30569.	25.	12.	148.	97.	268.	4445.	293.	6014.
13.	10485.	b/	0.	0.	23.	99.	1490.	99.	2143.
14.	14135.	5.	2.	29.	21.	64.	1236.	69.	1273.
15.	13367.	1.	7.	50.	31.	96.	1067.	97.	4280.
16.	18324.	10.	8.	83.	46.	137.	2383.	147.	5421.
17.	41659.	23.	3.	186.	93.	288.	4576.	311.	8126.
18.	20238.	35.	10.	130.	58.	200.	4115.	235.	6796.
19.	19883.	28.	5.	187.	77.	292.	5678.	320.	5317.
20.	16325.	17.	5.	104.	41.	155.	3581.	172.	3568.
Mean	17271.902	19.150	6.000	104.350	51.800	174.800	3064.350	193.950	4247.199
Standard Deviation	8206.066	12.558	5.487	58.475	26.305	85.675	1458.817	90.230	2008.870

Table 13 - List of Data, continued. . .

PRIVATE INSTITUTIONS

Case	Capital	Admin	BA	MA	PHD	FTE Faculty	FTE Enrollm	Labor	Other Expend a/
1.	3694.	18.	12.	49.	21.	89.	1270.	107.	2794.
2.	29505.	22.	17.	130.	92.	260.	3073.	282.	9298.
3.	5124.	13.	6.	40.	24.	72.	1219.	85.	2235.
4.	1895.	15.	0.	39.	19.	60.	803.	75.	714.
5.	7784.	9.	0.	75.	45.	79.	1227.	88.	3329.
6.	8989.	15.	1.	74.	17.	94.	1524.	109.	2653.
7.	7830.	13.	5.	46.	26.	89.	1118.	102.	3812.
8.	6984.	32.	3.	65.	29.	106.	1554.	138.	4121.
9.	6579.	26.	0.	26.	15.	48.	747.	74.	1579.
10.	5786.	1.	2.	37.	17.	59.	750.	60.	2772.
11.	7145.	15.	1.	44.	18.	64.	582.	79.	1503.
12.	7387.	50.	4.	46.	21.	74.	1061.	124.	3758.
13.	8372.	27.	1.	53.	27.	62.	1284.	89.	3730.
14.	9297.	8.	1.	27.	15.	47.	1291.	55.	2620.
15.	3090.	16.	6.	35.	10.	53.	456.	69.	562.
16.	9368.	26.	1.	46.	32.	80.	1487.	106.	3347.
17.	4661.	25.	2.	32.	13.	51.	921.	76.	2440.
18.	14140.	43.	0.	57.	40.	101.	1561.	144.	4380.
19.	3691.	2.	0.	38.	18.	48.	717.	50.	2044.
20.	5165.	7.	1.	28.	9.	44.	645.	51.	1938.
21.	4291.	5.	0.	32.	8.	43.	510.	48.	1185.
22.	30524.	2.	4.	135.	53.	196.	2587.	198.	8075.
Mean	8695.465	17.727	3.046	52.455	25.864	82.682	1199.409	100.409	1199.409
Standard Deviation	7401.371	12.855	4.259	29.324	18.722	51.671	636.894	54.270	636.894

a/ = 000's deleted
b/ = Not available.

Table 14

Zero-Order Correlation Matrices

	PHD	MA	BA	FTE FACULTY	ADMIN	LABOR	CAPITAL	OTHER EXPEND	FTE ENROLLM	
PHD	1.000	.907	.431	.953	.308	.948	.833	.784	.889	
MA		1.000	.484	.951	.452	.966	.683	.771	.937	PUBLIC
BA			1.000	.535	.040	.514	.113	.591	.513	
FTE FACULTY				1.000	.299	.991	.711	.782	.950	
ADMIN					1.000	.423	.163	.137	.370	
LABOR						1.000	.697	.762	.953	
CAPITAL							1.000	.735	.658	
OTHER EXPEND								1.000	.767	
FTE ENROLLM									1.000	
PHD	1.000	.871	.584	.923	.138	.911	.853	.900	.894	
MA		1.000	.493	.938	-.015	.890	.899	.892	.900	PRIVATE
BA			1.000	.682	.083	.669	.444	.552	.565	
FTE FACULTY				1.000	.083	.972	.913	.929	.935	
ADMIN					1.000	.316	.024	.151	.148	
LABOR						1.000	.875	.920	.926	
CAPITAL							1.000	.930	.903	
OTHER EXPEND								1.000	.942	
FTE ENROLLM									1.000	

--continued--

Table 14 - Zero-Order Correlation Matrices, continued. . .

	PHD	MA	BA	FTE FACULTY	ADMIN	LABOR	CAPITAL	OTHER EXPEND	FTE ENROLLM
PHD	1.000	.915	.552	.956	.226	.951	.876	.818	.899
MA		1.000	.543	.959	.259	.958	.794	.773	.936
BA			1.000	.626	.074	.612	.359	.601	.562
FTE FACULTY				1.000	.201	.990	.832	.799	.954
ADMIN					1.000	.338	.108	.155	.245
LABOR						1.000	.815	.790	.952
CAPITAL							1.000	.830	.778
OTHER EXPEND								1.000	.729
FTE ENROLLM									1.000

PUBLIC + PRIVATE

-36-

NOTES

1. For a discussion of financial distress in higher education, see Cheit (1971, 1973), Jellema (1971, 1973), Bowen and Minter (1974, 1975, 1976), National Commission on the Financing of Postsecondary Education (1973), and Lupton et al. (1976). For various policy recommendations for increased public and private financial support, see Carnegie Commission (1973, 1976), and the American Association of Colleges (1974).

2. Chronicle of Higher Education (February 22, 1977) and National Center for Education Statistics (1976).

3. The black land grant program of the federal government dates from 1890. The Developing Institutions Program (Title III of the Higher Education Act of 1965, as amended) began with a focus of bringing black colleges into the "mainstream" of American higher education; see Hodgkinson (1974) and Jacobs (1977). In 1971, the Ford Foundation announced a $100 million six year program to strengthen black colleges ($50 million) and encourage ethnic minority scholars ($50 million).

4. See Jones and Weathersby (1977) for a review of the financing of historically black colleges.

5. Student college going decisions can be considered as investments in human capital as pioneered by Becker (1964) and reviewed by Witmer (1971) and Freeman (1976), or as manifestations of student demand in a consumer theory framework, as reviewed by Jackson and Weathersby (1975).

6. The social returns to educational investments in the human capital model are discussed by Freeman (1976); the income distributional effects of education are analyzed by Hansen and Weisbrod (1969) and Jencks, et al. (1972); the impact of education on economic growth was highlighted by Denison (1967); and the impacts on student access and choice of financial aid are analyzed using a student demand formulation by Carlson, et al. (1974).

7. The application of microeconomic theory of the firm to higher education is reviewed by several authors in Wallhaus (1975).

8. The student achievement oriented literature is large. Some representative articles are Astin (1968), Coleman (1966), Mincer (1974), Toombs (1973), and Wise (1975). See Hanushek (1975) for a review of these concepts.

9. The application of consumer theory to student-faculty interactions is made by Kirschling and Staaf (1975).

10. Henderson and Quandt (1971) or Ferguson (1972) provide expositions of the neoclassical theory of the firm.

11. The neoclassical equilibrium conditions equate relative prices with relative marginal productivity of inputs for allocative efficiency and relative marginal utilities with the marginal rate of substitution along the production efficient frontier for preference efficiency. Second order equilibrium conditions insure convexity in the proper direction for each surface.

12. Cohen and March (1974) present their view of a college or university as an "organized anarchy" for which behavioral theories of for-profit firms are wholly inappropriate.

13. Silber (1976) and the American Association of Colleges (1974) are two advocates of a "high competition" world view.

14. Jackson (1977) found in his analysis of the National Longitudinal Study of the High School Class of 1972 that 61% of students who applied for admission to college applied to only one school, that 91% of all students who apply to college are admitted by their first choice school and that 95% of all applicants are admitted by at least one of their choices.

15. Willett (1976) found in her survey of 68 colleges, universities, and proprietary schools that one-third did not reply to a student inquiry, most provided scanty and misleading information, and only one school sent a second reply to encourage student interest.

16. Cootner (1974) discusses the differential pricing question and Lawrence, et al. (1970), Micek and Wallhaus (1973) and Wallhaus (1975) discuss the outcomes of higher education and the problems of output definition, measurement, valuation, jointness, etc.

17. Van Alstyne and Coldren (1976) and Bok (1977).

18. The linear production function does not meet the usual assumptions of declining marginal productivity of inputs and of a positive mixed second partial derivative of output with respect to multiple inputs; all second derivatives are zero. However, the linear function is useful to: (1) explore the data; (2) compare unit cost and linear regression analyses utilizing average rather than frontier behavior; and (3) provide a reference point to determine the degree of non-linearity.

19. Cobb and Douglas (1928), reviewed in Brown (1966) and Nerlove (1965).

20. Arrow, et al. (1961) are usually cited as originating the use of the CES function. See Walters (1963) and Brown (1966) for an extensive review and Biolley (1973) for a survey of recent developments.

21. Carlson (1976) cites over 90 references in his survey. There are numerous examples of unit cost studies completed since 1900 and they are surveyed in Cavanaugh (1969). Examples of the regression studies are Blau's (1973) study of college and university organization and administration in 115 institutions and Verry and Davies' (1976) analysis of British universities and polytechnic institutes. Bowen and Douglas' (1971) study of efficiency in liberal arts colleges is based on their assumed production function.

22. In profit-seeking firms one argues that to the extent the currently optimal production technology is known and available, one should expect all institutions to operate on the production efficiency surface, the frontier. With the conditions prevailing in colleges and universities that limit the fulfillment of assumptions described in Section II, there is no reason to believe all or most institutions are on the frontier.

23. Carlson (1975, 1972) briefly discusses the merits of each approach and describes in considerable detail the linear programming approach he uses. The linear programming approach was originally suggested by Farrell (1957), extended by Boles (1967, 1971) and Carlson (1972) and applied by Carlson (1972) and in a slightly different form, by Levin (1974).

24. Carlson (1975) gives the following mathematical formulation of the linear programming problem:

> Let X_{it} = the quantity of the i^{th} resource used by the t^{th} institution, Y_{jt} = the quantity of the j^{th} outcome at the t^{th} institution, and C_{kt} = the k^{th} characteristics variable for the t^{th} institution, then the general linear programming model to determine the frontier-production relationships for T institutions with N resources, M outcomes, and Q characteristic variables is written as:

$$\text{Max} \sum_{t=1}^{T} z_t Y_{rt}$$

$$\text{subject to:} \sum_{t=1}^{T} z_t X_{it} \leq X_{is} \qquad i = 1, \ldots, N$$

$$\sum_{t=1}^{T} z_t Y_{jt} \geq Y_{js} \qquad j = 1, \ldots, r-1, r+1, \ldots, M$$

$$\sum_{t=1}^{T} z_t Y_{rt}(C_{kt} - C_{ks}) \leq 0 \qquad k = 1, \ldots, Q_1$$

$$\sum_{t=1}^{T} z_t Y_{rt}(C_{kt} - C_{ks}) \geq 0 \qquad k = Q_1 + 1, \ldots, Q$$

$$z_t \geq 0 \qquad t = 1, \ldots, T$$

25. This approach assumes a convex production function, consistent with decreasing marginal productivity of inputs, and otherwise makes no assumptions of the shape of the convex hull. It is sensitive to data errors and other sources of extreme outlying data points. Timmer (1971) developed a modified Farrell technique which discards a predetermined percent of extremely efficient data points as one means of avoiding the possibly greatly misleading effects of a small number of data errors.

    Another issue is that Farrell (1957) suggests a special case in which the hyperplanes estimated all go through the origin, assuming linear homogeneity of degree 1. Carlson's (1976) analysis does not make this assumption and the sides of the convex hull should be identified as well as the segment through the origin.

26. See Jacobs (1977) and Harvard Graduate School of Education (1977) for a discussion of the federal policy towards financially supporting black colleges.

27. See Jones and Weathersby (1977) and Browning (1974) for a discussion of the role of philanthropy in financially supporting black colleges.

28. Adams vs. Richardson, 356 F supp. 92(DDC 1973).

29. See Willie (1977) for a compendium of recent research on black colleges and Bowles and DeCosta (1971) for a description of black colleges.

30. See Jackson (1977b, data appendix) for a description of the entire data base available. The variables actually used are described in the text.

31. Astin (1968, 1973), Chickering (1975), Hanushek (1975), Jencks, et al. (1972), Levin (1974) are examples of student change as the measure of college outcomes.

32. This road is "less travelled by" but is implicit in the work of Carlson (1972, 1975, 1976), Bowen and Douglas (1971), and most of the descriptive cost analyses.

33. This raises the question of whether this and previous studies are identifying a demand or a supply relationship when student enrollment is used as a proxy for institutional outcomes.

34. The linear programming frontier identification approach incorporates multiple products directly. Relatively few estimations of continuous production functions have included the simultaneous effects of multiple outcomes; see Verry and Davies (1976, pp. 199-201).

35. We experimented with the composition of enrollment--part-time, full-time, percent low income, percent ethnic minority, percent in top quarter of high school graduating class--but these were all headcount and not full-time equivalent measures and showed little stability.

36. It is unclear whether the lack of support staff data will lead to an overestimate of labor productivity, which would occur if there is a high correlation between the number of support staff and our total labor variable. The frontier was identified using total dollar expenditures and, therefore, omitting support staff may introduce some specification error with or without parameter bias.

37. Although colleges can use any of several acceptable valuation methods, most use book value or the price of assets at the time they were acquired. The differential age of colleges can mean that long term price inflation is confounded with differential capital stock measures. In any event, we would expect the book value to understate the current value of permanent assets and overstate the depreciated value of equipment.

38. Ringstad's (1970) comments on production function estimation in manufacturing are appropriate here:

> . . .there is a need for better theories concerning the behavior of the production units, and more than anything else there is a need for better data. Limitations in the data are likely to be the more efficient constraint in the development of production function estimation. . . . We must . . .try to determine which weaknesses are the most serious ones. . . . Probably the most efficient way of doing this is just to carry out econometric studies on such data.

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